Ephemeris Interpolation Techniques for Assisted GNSS Services

Matteo Iubatti*, Marco Villanti*, Alessandro Vanelli-Coralli*,
Giovanni E. Corazza*, Stephane Corazza**

* DEIS/ARCES - University of Bologna - Viale Risorgimento 2 - 40136 Bologna, Italy,
Tel: +39 051 2093056, Fax: +39 051 2093053,
e-mail: {miubatti, mvillanti, avanelli,gecorazza} @ deis.unibo.it
** Alcatel Alenia Space France - Avenue J.F. Champollion 26, 31037 Toulouse, France,
Tel: +33 (0)534 354281, Fax: +33 (0)534 354943,
e-mail: Stephane.Corazza@alcatelaleniaspace.com
Ephemeris, Interpolation techniques, A-GNSS, assistance services.

Abstract. Assisted-GNSS (A-GNSS) is an interesting technology that can consistently improve positioning performance and reduce terminal complexity. One of the possible data that the A-GNSS can provide is the satellite orbit (satellite ephemeris) necessary to determine the user position starting from pseudorange measurements. In this paper, we propose two novel data structures for the transmission of ephemeris through the assistance network. It is shown that, adopting different interpolation techniques, it is possible to consistently reduce the amount of data to be transmitted and extend the ephemeris validity to 24 hours.

1 Introduction

The European Galileo program is devoted to the development of a satellite based positioning system that will provide enhanced accuracy and continuous reliability to civilian users. To enhance performance and compatibility, Galileo has been designed to be possibly interoperable with the American Global Positioning System (GPS), so that the two systems can cooperate in synergy and exploit a larger satellite constellation. Further, in order to guarantee efficient positioning and navigation services in critical environments, both Galileo and GPS can be improved by assistance services supported by terrestrial networks. With this strategy, identified as Assisted Global Navigation Satellite Services (A-GNSS), users can benefit of assistance data to perform positioning and navigation more efficiently and quickly, for example by reducing the time-to-first-fix (TTFF) in code acquisition. The potential benefits of A-GNSS on terminal complexity reduction and performance improvement have stimulated the research community towards the definition and standardization of different data aiding sets. Among many others, the transmission of ephemeris (i.e. the satellite orbit description parameters) through the terrestrial aiding network can be very effective to reduce time-to-first-fix onto satellite signals because the receiver is released from the task of receiving it from the satellite GNSS network, which has low bitrate and suffer from adverse propagation environments, such as in urban areas.

Because the accurate computation of the satellite position is a fundamental prerequisite in all GNSS positioning systems, particular attention must be taken in defining the ephemeris assistance field. A possible solution is to adopt for the assistance ephemeris field the same structure of broadcast ephemeris, which are described in the navigation message by a set of parameters to be employed in an accurate equation describing the satellite orbit, as detailed in the Galileo navigation model reported in [1]. In order to reduce the amount of data to be transmitted while preserving high accuracy, ephemeris data are valid for a time interval, which however is limited so that they need to refreshed frequently. For GPS, ephemeris are updated by the control center every 2 hours, to allow an accuracy of the computed coordinates in the order of about 3 meters, while for Galileo it is foreseen that the ephemeris will be updated every 3 hours. Also, an overlapping period is foreseen in order to prevent possible gaps, as depicted in Fig. 1.

Note that this very accurate orbit description is necessary for high-precision services, while less stringent constraints are adequate for mass market services. In fact, in low cost terminals the pseudorange estimation can be performed with a limited accuracy so that a satellite position error in the order of a few meters becomes negligible. For example, an error in the satellite coordinates of 20m is equivalent to having pseudorange estimation errors in the order of 0.1μsecond, which is a reasonable precision for mass market terminals.

Starting from this observation, the aim of this paper is to investigate new efficient solutions for ephemeris description, in order to extend the data validity with respect to the standard GNSS approach and/or to reduce the amount of data to be

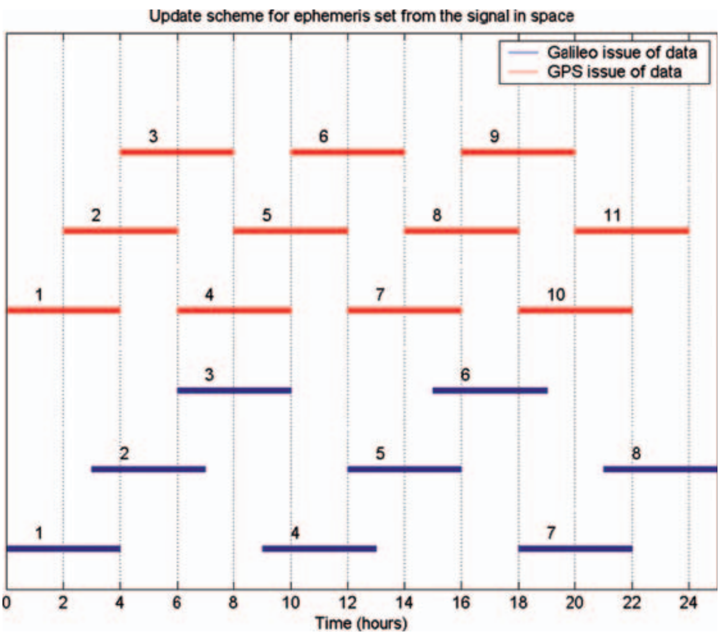

Fig. 1. Galileo and GPS ephemeris issues over 24h.

transmitted, under the constraint of preserving a precision for the satellite orbit description in the order of 20m. Two alternatives are identified in this paper: the first foresees transmitting the samples the satellite coordinates derived at a low sampling rate, and then interpolating these in the receiver to obtain the satellite position with the desired accuracy; the second envisages the transmission of the coefficients of a simplified function describing the satellite orbit. In particular, the idea is to find a polynomial function that properly describes the ephemeris in order to transmit the polynomial coefficients instead of the satellite position points. Notably, both techniques are based on interpolation, so that it is essential to find the best interpolation techniques to make these alternative approaches effective. To this aim, different alternatives are considered to find the best trade-off between accuracy and bit requirement.

2 Ephemeris Interpolation Techniques

Interpolation is a method of constructing new data points from a discrete set of known data points. Typically, this is achieved by sampling a function which closely fits the known data set. The process of identifying the approximating function is generally called curve fitting, and interpolation is a specific case of curve fitting in which the function must go exactly through the known data values.

The simplest way of interpolation is the linear interpolation, in which the known points are connected by segments. Linear interpolation has a very limited complexity but does not provide high accuracy, so that alternative methods have been developed. A generalization of linear interpolation is given by polynomial interpolation, which encompass a family of different strategies. An example of polynomial interpolation is the Lagrange interpolation, which is one of the most widely adopted interpolation techniques and considers a polynomial of degree $n - 1$ going through all the n known data values. The interpolation error is proportional to the distance between the data points to the power n. Although polynomial interpolation consistently improves the linear alternative, it introduces oscillations in the region close to the border of the known data set that make the interpolation very poor in these regions: this effect is known as Runge's phenomenon. Also, it has to be noted that the evaluation of the polynomial interpolation is computationally demanding with respect to linear interpolation. The polynomial interpolation family also includes another well known approach called trigonometric (or Fourier) interpolation, which results to be especially suitable for the interpolation of periodic functions. In this case, the interpolant is given by the sum of sines and cosines of given periods. An important special case is when the given data points are equally spaced and the exact solution is given by the discrete Fourier transform [4].

The disadvantages of polynomial interpolation can be limited by using spline interpolation. The spline interpolation uses low-degree polynomials to approximate each data set segment, selecting the polynomial pieces such that they fit smoothly together. The resulting function is called spline. For instance, the natural cubic spline is piecewise cubic and twice continuously differentiable. Like polynomial interpolation, spline interpolation incurs a smaller error than linear interpolation and the interpolant is smoother. Additionally, the interpolant is easier to evaluate than the high-degree polynomials used in polynomial interpolation and it is also able to limit the Runge's phenomenon. The interpolation methods briefly described above are detailed in the following, reporting the achievable interpolation performance when processing actual ephemeris data. In particular, the analysis reported in the following has been conducted using GPS ephemeredes that are given at 900 sec (15 min) in accordance with *.sp3 file format defined in 1991 by Remondi [2, 3].

2.1 Lagrange Polynomial Interpolation

Given the $n + 1$ ephemeris values $f(t_0), \ldots, f(t_n)$ at the distinct times $t_0, \ldots, t_n$, it exists an unique interpolating polynomial $p_n(t)$ satisfying the condition

$$p_n(t_i) = f(t_i) \text{ for } i = 0, \ldots, n \tag{1}$$

The polynomial $p_n(t)$ is usually identified as Lagrange interpolation polynomial and can be written in the form

$$p_n(t) = \sum_{i=0}^{n} f(t_i)\, l_i(t) \tag{2}$$

where the polynomial coefficients are given by

$$l_i(t) = \frac{\prod_{k=0}^{i-1}(t - t_k)}{\prod_{k=0}^{i-1}(t_i - t_k)} \cdot \frac{\prod_{k=i+1}^{n}(t - t_k)}{\prod_{k=i+1}^{n}(t_i - t_k)} \tag{3}$$

Several evaluations of the accuracy of this method has been made, and it is generally found that an 8-th order Lagrange polynomial interpolation is able to extrapolate data in the center of 8 points spaced by 900 sec with a precision of 1 cm [6]. Unfortunately, this high level accuracy is not reached when the entire set of ephemeris data, covering 24 hours, is interpolated. For example, in Fig. 2 it is reported the Euclidean distance between the interpolated points and the precise ephemeris of the 7th and the 23rd order Lagrange polynomial obtained by using a subset of 8 and 24 values, respectively, extracted by down-sampling the set of 96 precise ephemeris values (one every 900s) describing the orbit for the entire day. It can be seen that the distance between the interpolated values and the exact ones is very large for the 7th order polynomial because the spacing between the input data is too large to be compensated. Differently, the interpolating accuracy is more acceptable for the 23rd order polynomial, at least out of the Runge's phenomenon region. Obviously this higher precision comes at the price of increased complexity.

To reduce the Runge's phenomenon, the k-th order Lagrange method is usually adopted with successive intervals that overlap in time [6].

2.2 Trigonometric (Fourier) Polynomial Interpolation

Differently from Lagrange method that is a standard interpolation technique typically used for continuous differentiable functions defined on compact intervals, the trigonometric (or Fourier) polynomial interpolation is particularly suited when the

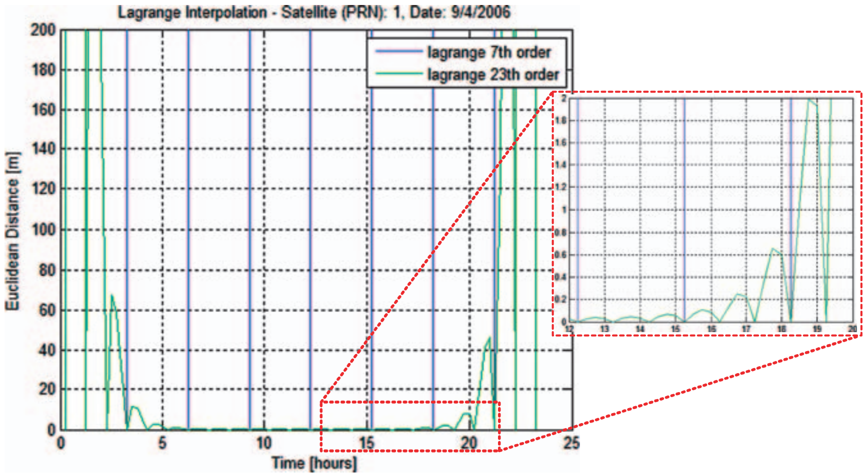

Fig. 2. Euclidean distance between the interpolated pointsand the precise ephemeris of the 7th and the 23rd order Lagrange polynomial.

data set to be interpolated has a clear periodic trend. This is exactly the case of ephemeris data, which results to be almost periodic, with a period of 24 hours if the satellite coordinates are referred to a Earth-centre, Earth-fixed Cartesian coordinate system, as reported in Fig. 3 [5] [6]. The Fourier interpolation approach is based on the idea to assume an interpolating function defined over the interval [0, 2π], defined as

$$p_n(t) = a_0 + \sum_{k=1}^{n} (a_k \cos(\omega t) + b_k \sin(\omega t)) \tag{4}$$

Considering the ephemeris periodicity, we can restrict our attention to a single 24 hour period and generate a trigonometric polynomial using all data available over that period. In fact, the complexity of the method is given by the order of the polynomial function and not by the number of points to be interpolated. In particular, because the satellite orbit is not truly periodic, the polynomial coefficients, including ω, are iteratively obtained by minimizing the error between the interpolated and the exact satellite positions. For this reason, the interpolation accuracy increase by considering a larger known data set. Since the error incurred by assuming the data to be periodic over a k day period would be almost k times greater than the error incurred from assuming the orbit to be periodic over a single day, we have to adopt this approach over intervals that do not exceed the fundamental period (24 hours) of the ephemeris data [6]. The precision that can be achieved by the trigonometric interpolation is reported in Fig. 4 in terms of Euclidean distance between the interpolated and the precise ephemeris for the 5th and 8th order on a 24 hour period. It can be seen that the 8th order polynomial provides a very good approximation, by providing an error lower than 20 m for more than 18-20 hours per day, thanks to a limited Runge's effect, which makes the approximation poor only at the border of the data set.

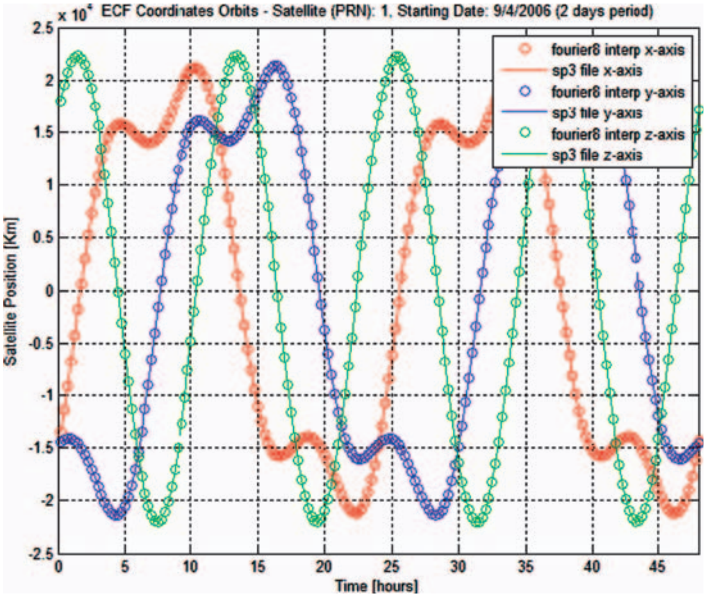

Fig. 3. Ephemeris and fourier interpolated values for a 2 days period. Notice that the function is quasi-periodic.

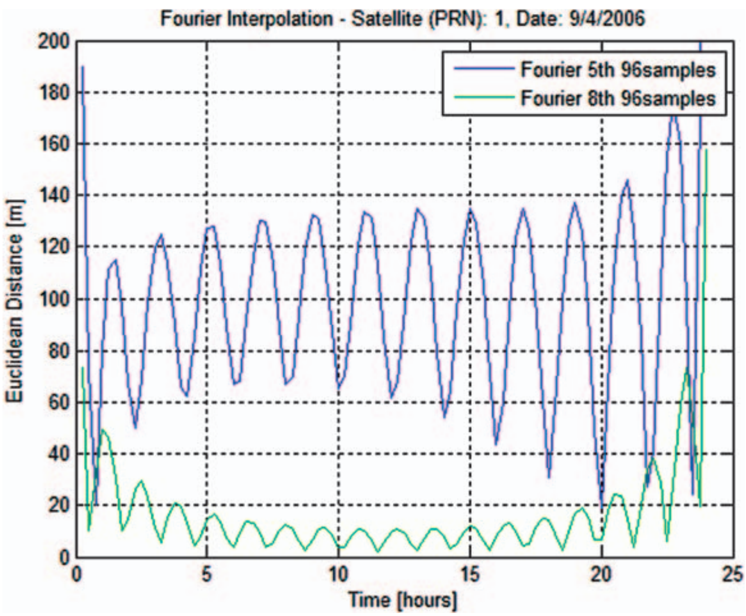

Fig. 4. Fourier 5th and 8th order polynomial interpolation precision in terms of euclidean distance between exact and interpolated values for 1 day period.

2.3 Spline Interpolation

The spline interpolation is an approach that interpolates using a function piecewise defined by polynomials. The spline interpolation is often preferred to polynomial interpolation because it yields similar results, even when using lower degree polynomials, so reducing the Runge's phenomenon characteristic of higher degrees. Additionally, spline is computationally efficient and has an advantage with respect to Lagrange interpolation because it allows to calculate the interpolating polynomials over the entire interval only a single time, at the beginning of the interpolation process. This calculation involves solving a system of equations each of degree k [6].

In its most general form, a polynomial spline $S(t)$: $[a,b] \rightarrow \mathbb{R}$ consists of polynomial pieces P_i: $[t_i, t_{i+1}] \rightarrow \mathbb{R}$, where $a = t_0 < t_1 < \cdots < t_{k-1} < t_{k-2} = b$. The given k points t_i are called knots. If the knots are equidistantly distributed in the interval $[a, b]$ we say the spline is uniform, otherwise we say it is non-uniform.

Given $n + 1$ distinct knots t_i with $n + 1$ knot values y_i the spline interpolation finds an interpolant of degree n as

$$S(t)=\begin{cases} S_0(t) \rightarrow t \in [t_0, t_1] \\ S_1(t) \rightarrow t \in [t_1, t_2] \\ \quad \cdots \\ S_{n-1}(t) \rightarrow t \in [t_{n-1}, t_n] \end{cases} \tag{5}$$

where each $S_i(t)$ is a polynomial of degree k, which identifies the spline order (for example quadratic spline adopts $k = 2$, cubic spline $k = 3$, etc.). The unique interpolant $S(t)$ is obtained applying boundary conditions between different intervals calculating derivates in the discontinuity points. For our study we have adopted two different spline interpolation functions embedded in MatLab 7.0 called *csape* and *spapi.* The former, *csape* implements a cubic spline interpolation with the possibility to insert a condition to process the end points of the data set in order to reduce border effects (in this deliverable we have used default as ending condition). Differently, *spapi* is the traditional spline interpolation of kth order. Note that, the fact that each subinterval is represented by a kth order polynomial (where $k < n$, in general) means that the evaluation on each interval is much quicker than the Lagrange n-th order counterpart. Obviously, if we process a great set of data, the dimension of the system of equations increase at the price of larger computational complexity. In Fig. 5, the Euclidean distance between the interpolated and precise ephemeris is reported for spline interpolation with 3, 8 and 24 knots, respectively. It is possible to note that only using 24-th order polynomial we can reach a precision in order of few centimeters for one day validity, although the Rounge effect limits the validity of the interpolation accuracy.

3 Ephemeris Data Definition for Assistance Service

The different interpolation techniques discussed in the previous section have positive and negative aspects. In particular, Lagrange and spline of high order are able to ensure a very large accuracy, at least out of the Rounge region. This results is very

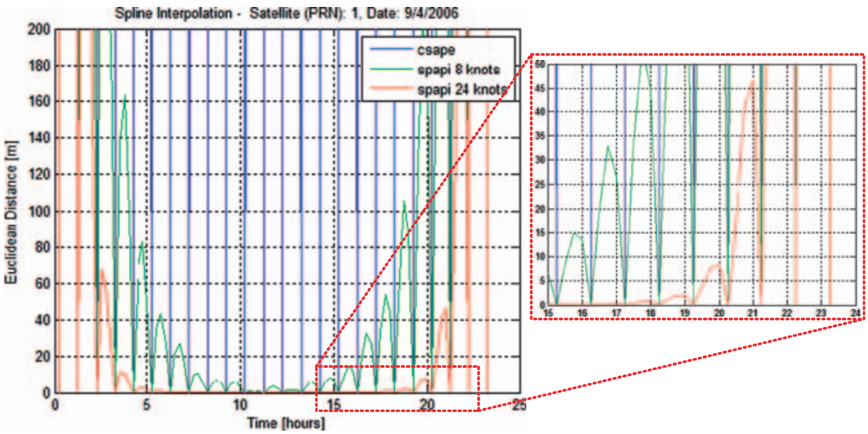

Fig. 5. Cubic, 8th and 24th order spline interpolation precision in terms of Euclidean distance between real data and interpolated results for 24 hours validity.

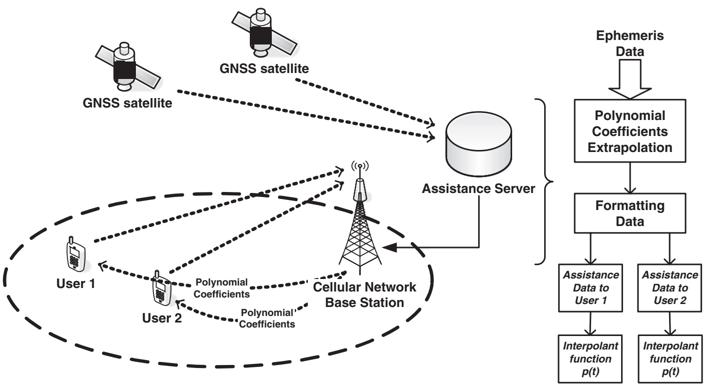

Fig. 6. Assistance GNSS network architecture for the proposed ICT (Interpolation Coefficients Transmission) method.

interesting even though a large accuracy requires a large number of coefficients to describe the interpolation function. Conversely, the trigonometric polynomial interpolation is characterized by a more limited accuracy, but the interpolation function can be described by a more limited number of coefficients. The different characteristics of the proposed interpolation methods can be exploited to provide two alternative solutions for the ephemeris data definition, which are identified as Interpolation Coefficient Transmission (ICT) and Satellite Coordinates Transmission (SCT), respectively. In the ICT solution, precise ephemerides are processed by the assistance server, which then broadcasts the interpolation function coefficients opportunely formatted, as depicted Fig. 6. Because the Fourier interpolation provides, a good trade-off between precision (within 20 m for more than

Table 1. Fourier 8th order polynomial coefficients for 4 GPS satellite orbits on 09/04/2006.

PRN 1	a0	a1	b1	a2	b2	a3	b3	a4	b4	a5	b5	a6	b6	a7	b7	a8	b8	w
x	0.018595	−10391	17641	−0.19771	0.084445	−5000.6	3137.2	0.10498	−0.00846	18.433	1.3311	0.023298	0.003296	−0.02002	−0.01322	0.011524	0.003335	0.065635
y	0.63236	−17918	−10391	−0.96537	−1.1133	3186.2	5119	0.20918	0.30531	1.3964	−18.553	0.040973	0.047285	0.011532	0.061515	0.01783	0.015918	0.065638
z	204.86	−0.14608	0.010998	15908	15441	0.07049	0.000688	−11.664	−67.982	0.021555	0.005715	−0.01101	0.14478	0.008354	0.001515	0.004906	0.001466	0.065636
PRN 2	a0	a1	b1	a2	b2	a3	b3	a4	b4	a5	b5	a6	b6	a7	b7	a8	b8	w
x	0.14776	12823	−16714	−0.08337	0.36025	21.191	5656.4	−0.04442	0.026425	−18.731	−18.113	−0.02298	0.034367	0.13347	0.062799	−0.0176	0.028182	0.065637
y	−0.13186	16893	12361	−0.13135	−0.05226	5466.2	9.7661	0.15503	0.005604	−17.236	17.926	0.037545	0.003117	0.052779	−0.14424	0.020019	0.001771	0.065638
z	−267.61	0.066031	−0.04781	−6820.2	20508	−0.02451	0.034587	−45.706	−87.852	−0.00653	−0.0063	0.51226	0.28713	−0.00322	−0.00193	−0.00655	−0.00272	0.065637
PRN 3	a0	a1	b1	a2	b2	a3	b3	a4	b4	a5	b5	a6	b6	a7	b7	a8	b8	w
x	0.024956	−15898	14221	−0.14739	−0.02449	−4551	2563.9	0.049048	0.005681	18.399	−9.345	0.010413	0.001602	−0.09515	0.09312	0.004917	0.001827	0.065636
y	−0.65332	−14513	−15424	1.0555	1.7146	2676.6	4678.4	−0.24613	−0.53046	−10.39	−19.191	−0.0422	−0.09217	0.076747	0.046322	−0.0171	−0.03974	0.065633
z	−155.73	0.14136	−0.0018	−12499	−17172	−0.07433	−0.00992	47.445	69.896	−0.01378	−0.00684	−0.44138	−0.37007	−0.00766	−0.00224	−0.00049	−0.00023	0.065636
PRN 4	a0	a1	b1	a2	b2	a3	b3	a4	b4	a5	b5	a6	b6	a7	b7	a8	b8	w
x	−1.0986	3322.6	−21015	−1.7921	−7.5443	2384.2	4946.9	1.567	2.4124	9.3026	18.944	0.42526	0.50447	0.3583	0.44697	0.20468	0.22337	0.065656
y	2.2072	20447	3430.7	−1.1419	−0.60776	5095	−2370.7	−0.45789	−0.06617	18.918	−8.645	−0.16115	−0.07136	0.030598	−0.13895	−0.07969	−0.05707	0.065639
z	−35.729	0.068564	0.00279	3034.2	21391	−0.0138	0.001118	10.451	78.965	−0.00404	−0.00245	0.11857	0.58655	−0.00274	−0.00073	−3.88E−05	0.005408	0.065641

18 hours per day) and bit requirements (18 coefficients for the 8th order solution), it represents the best solution to make the ICT method effective.

In Table 1, an exemplary coefficient set is provided for 4 GPS satellites (PRN1 to PRN4) orbits on 9/04/2006.

Each interpolation function coefficient can be inserted in the navigation message with a 32-bit occupation and this implies a total length of 1728 bits for the ephemeris assistance package, as reported in Table 2. The assistance message length required by Fourier interpolation approach to describe the satellite orbits is greater than the conventional technique but its larger time validity makes this solution interesting for assistance purpose.

Note that, the coefficients of Table 2 have been by rounding the original double format numbers processed by MatLab, in order to have a limited number of decimals. The effect of this truncation is reported in Fig. 7, where it is reported the Euclidean distance between the exact interpolation function and the approximated version, obtained with a reduced number of bit for the coefficients representation. It can be seen that the use of a reduced accuracy coefficients (with only a few decimal digits) do not introduce a significant precision loss in comparison with the case of the exact 32 floating points coefficient description. This is a very interesting starting point in order to reduce the total assistance message length. In particular, it can be advantageous to employ a fixed point representation for each coefficient, optimizing the number of digits separately. Even though this activity is left for future investigations, it is possible to foresee that the assistance message length can be consistently shortened with respect to the value indicated in Table 2, which can be seen as an upper bound.

Table 2. Bit occupation and time validity for traditional and ICT assistance methods.

Ephemeris assistance approach	Required fields in the assistance message	Bit occupation in the assistance message for 4 hours validity	Bit occupation in the assistance message for 24 hours validity
Traditional Approach (GPS)	15 (for the 3D orbit)	362 (only spatial coordinates)	–
	$15 \times 6 = 90$ (6 retransmission for 24h)	–	2172 (only spatial coordinates)
	$15 \times 12 = 180$ (12 retransmission per 24h, high reliability)	–	4344 (only spatial coordinates)
Traditional Approach (Galileo)	15 (for the 3D orbit)	362 (only spatial coordinates)	–
	$15 \times 8 = 120$ (8 retransmission for 24h)	–	2896 (only spatial coordinates)
Interpolation Coefficients Transmission (ICT)	$18 \times 3 = 54$ (for the 3D orbit)	–	**1728** (only spatial coordinates)

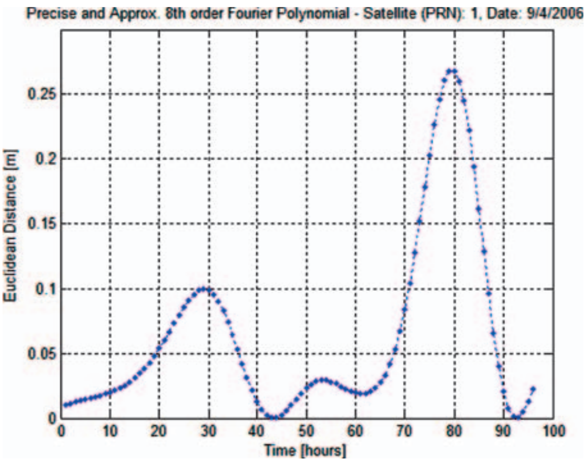

Fig. 7. Coefficient approximation impact for the Fourier polynomial.

Differently from ICT, SCT is based on the transmission of the satellite coordinates, sampled at regular intervals. The number of points to be transmitted to cover 24 hours depends on the desired accuracy and on the interpolation function adopted at the receiver side, as reported in Fig. 8.

In this case, the assistance message size is directly determined by the number of points composing the data set to be transmitted, which, however, strongly depends on the interpolation techniques adopted at the receiver side. To this purpose, Lagrange and spline interpolation can be fruitful employed, as explained in previous sections, although different terminal classes can employ different interpolation functions. To roughly quantify the amount of data to be transmitted, in Table 3 the minimal number of bit to be transmitted is reported for a 26 bit coordinates representation. This representation considers precise ephemeris rounded within 1 meter precision. The impairment of this approximation is quantified in Fig. 9, where the

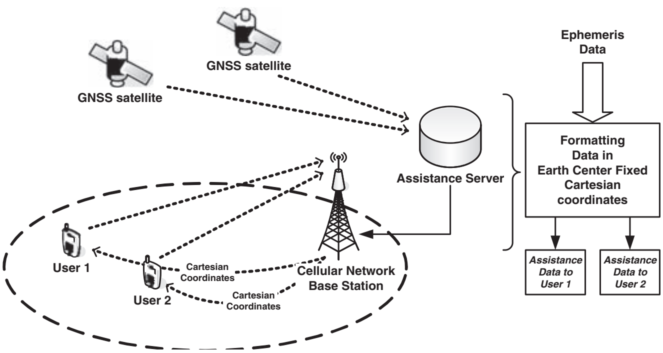

Fig. 8. SCT (Satellite Coordinate Transmission) system architecture for ephemeris assistance data.

Table 3. Bit occupation and time validity for traditional and SCT assistance methods.

Ephemeris assistance approach	Required fields in the assistance message	Bit occupation in the assistance message for 4 hours validity	Bit occupation in the assistance message for 24 hours validity
Traditional Approach (GPS)	15 (for the 3D orbit)	362 (only spatial coordinates)	–
	15 × 6 = 90 (6 retransmission for 24h)	–	2172 (only spatial coordinates)
	15 × 12 = 180 (12 retransmission for 24h, high reliability)	–	4344 (only spatial coordinates)
Traditional Approach (Galileo)	15 (for the 3D orbit)	362 (only spatial coordinates)	–
	15 × 8 = 120 (8 retransmission for 24h)	–	2896 (only spatial coordinates)
Satellite Coordinate Transmission (SCT)	24 × 3 = 72 (for the 3D orbit)	–	**1872** (only spatial coordinates)

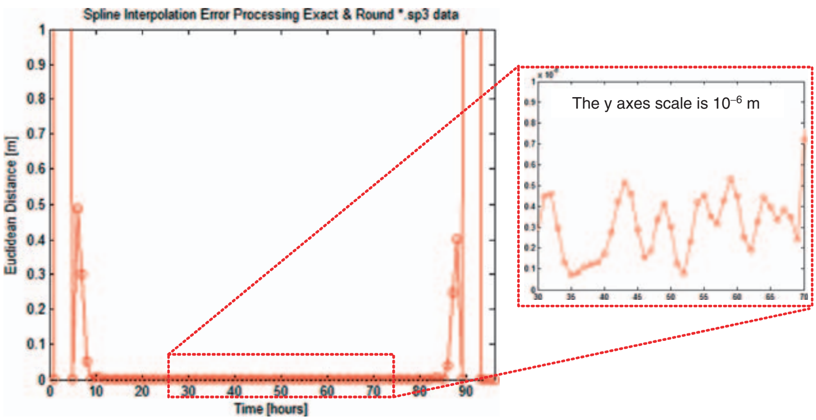

Fig. 9. Euclidean distance between exact ephemeris and rounded ephemeris spline interpolation.

Euclidean distance between precise ephemeris interpolation and approximate ephemeris interpolation is reported.

4 Conclusions

In this paper we have proposed two possible approaches to define novel data sets to describe satellite ephemeris for one day validity. The objective of this study has been the reduction of the number of bits required to describe the satellite orbit with respect to the solution adopted in satellite GNSS systems, under the constraint that the introduced error has to be lower than 20 m. This is in fact a reasonable approximation for most of commercial applications. The first solution, identified as Interpolation Coefficients Transmission (ICT), the A-GNSS server transmits the coefficients of a trigonometric interpolation polynomial that is employed by terminals to evaluate the satellite orbit with one day validity. In this case, the 8th order Fourier interpolation approach, which requires 18 coefficients to completely define the interpolation function, allows achieving the 20 m desired precision. The second approach, identified as Satellite Coordinates Transmission (SCT), foresees the transmission of a sampled version of precise ephemeris. It has been shown that terminals using a 24 knots spline interpolation method are able to largely meet the 20 m precision requirement with only 24 satellite position points with 26 bit-precision in one day. The main result is that both techniques allow an effective reduction of the number of bit to be transmitted to describe the satellite orbit, in a complementary way. In fact, ICT reduces terminal complexity because the interpolation processing is performed by the assistance server. Differently, SCT, which is more computationally demanding for terminals, introduces flexibility in the service provision, being the achievable precision depending on the terminal computational capability.

Acknowledgment

This work has been supported by the GJU GAMMA project (GJU 05/2413 CTR GAMMA).

References

[1] European Space Agency / Galileo Joint Undertaking, "Galileo Open Service Signal In Space Interface Control Document (OS SIS ICD) Draft 0", Draft 0, 23/05/2006.

[2] B. W. Remondi, "NGS Second Generation ASCII and Binary Orbit Formats and Associated Interpolation Studies", Proc. of the Twentieth General Assembly of the IUGG, 1991.

[3] M. Horemuz, and J. V. Andersson, "Polynomial interpolation of GPS satellite coordinates", GPS Solutions, vol. 10, n. 1, 2006, pp. 67–72.

[4] M. T. Heideman, D. H. Johnson, and C. S. Burrus, "Gauss and the history of the fast Fourier transform", IEEE ASSP Magazine, vol. 4, n. 1, 1984, pp. 14–21.

[5] M. Schenewerk, "A brief review of basic GPS orbit interpolation strategies", GPS Solutions, vol. 6, n. 4, 2003, pp. 265–267.

[6] B. Neta, C. P. Sagovac, D. A. Danielson, and J. R. Clynch, "Fast Interpolation for Global Positioning System (GPS) Satellite Orbits", Proc. of the AIAA/AAS Astrodynamics Conference, June 1996.

[7] P. Korvenoja, and R. Piché, "Efficient Satellite Orbit Approximation", Proc. of the ION GPS 2000, September 2000.

[8] D. J. R. Van Nee, and A. J. R. M. Coenen, "New Fast GPS Code-Acquisition Technique Using FFT", Electronics Letters, vol. 27, n. 2, 1991.